
Logarithme en base a

■ Propriétés

$$\forall x \in \mathbb{R}_0^+ : y = \log_a x \iff x = a^y$$

$$\forall x \in \mathbb{R} : \log_a(a^x) = x$$

$$\forall x \in \mathbb{R}_0^+ : a^{\log_a x} = x$$

■ Propriétés

$$\forall x, y \in \mathbb{R}_0^+$$

$$\blacksquare \log_a 1 = 0$$

$$\log_a 1 = \log_a a^0 = 0$$

$$\blacksquare \log_a a = 1$$

$$\log_a a = \log_a a^1 = 1$$

$$\blacksquare \log_a(x \cdot y) = \log_a x + \log_a y$$

on pose $u = \log_a x$ et $v = \log_a y$

on a alors $x = a^u$ et $y = a^v$

$$\log_a(x \cdot y) = \log_a(a^u \cdot a^v) = \log_a a^{u+v} = u + v = \log_a x + \log_a y$$

$$\blacksquare \log_a\left(\frac{1}{x}\right) = -\log_a x$$

on pose $u = \log_a x$

on a alors $x = a^u$

$$\log_a\left(\frac{1}{x}\right) = \log_a\left(\frac{1}{a^u}\right) = \log_a a^{-u} = -u = -\log_a x$$

$$\blacksquare \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a\left(x \cdot \frac{1}{y}\right) = \log_a x + \log_a \frac{1}{y} = \log_a x - \log_a y$$

$$\blacksquare \log_a x^n = n \cdot \log_a x$$

on pose $u = \log_a x$

on a alors $x = a^u$

$$\log_a x^n = \log_a (a^u)^n = \log_a a^{n \cdot u} = n \cdot u = n \cdot \log_a x$$

$$\blacksquare \log_a x = \frac{\log_b x}{\log_b a}$$

on pose $u = \log_a x$

on a alors $x = a^u$

$$\log_b x = \log_b a^u = u \cdot \log_b a = \log_a x \cdot \log_b a$$

$$\log_b x = \log_a x \cdot \log_b a$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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