

$$f(x) = \operatorname{Arctg}\left(\frac{x+1}{x}\right)$$

### 1. Domaine de définition

$$\operatorname{Dom} f = \mathbb{R} \setminus \{0\}$$

### 2. Signe de f

|  |   |    |   |   |   |
|--|---|----|---|---|---|
| $x$  |   | -1 |   | 0 |   |
| $\operatorname{Arctg}\left(\frac{x+1}{x}\right)$ | + | 0  | - |   | + |

### 3. Limites et asymptotes

$$\left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = -\frac{\pi}{2} \\ \lim_{x \rightarrow 0^+} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = \frac{\pi}{2} \end{array} \right.$$

$$\lim_{x \rightarrow +\infty} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = \frac{\pi}{4}$$

$$\lim_{x \rightarrow -\infty} \operatorname{Arctg}\left(\frac{x+1}{x}\right) = \frac{\pi}{4}$$

$$\text{AH} \equiv y = \frac{\pi}{4}$$

### 4. Intersection avec les axes

$$\operatorname{Gf} \cap X = \{(-1, 0)\}$$

$$\operatorname{Gf} \cap Y = \{\}$$

### 5. Etude de f'

$$f'(x) = -\frac{1}{2x^2 + 2x + 1}$$

|                        |   |   |   |
|------------------------|---|---|---|
| $x$                    |   | 0 |   |
| $-\frac{1}{2x^2+2x+1}$ | - |   | - |
| $f'(x)$                | ↘ |   | ↘ |

### 6. Etude de f''

$$f''(x) = \frac{2(2x+1)}{(2x^2+2x+1)^2}$$

|                                 |   |                  |   |   |   |
|---------------------------------|---|------------------|---|---|---|
| $x$                             |   | $-\frac{1}{2}$   |   | 0 |   |
| $\frac{2(2x+1)}{(2x^2+2x+1)^2}$ | - | 0                | + |   | + |
| $f''(x)$                        | — | $-\frac{\pi}{4}$ | — |   | — |

$$I : \left(-\frac{1}{2}, -\frac{\pi}{4}\right)$$

### 7. Tableau récapitulatif

2 |  $\arctg(x+1):x.nb$

|           |                     |   |      |   |                  |   |      |   |                     |
|-----------|---------------------|---|------|---|------------------|---|------|---|---------------------|
| $x$       | $-\infty$           |   | $-1$ |   | $-\frac{1}{2}$   |   | $0$  |   | $+\infty$           |
| $f(x)$    | $\frac{\pi}{4}$     | + | $0$  | - | $-\frac{\pi}{4}$ | - | $ $  | + | $\frac{\pi}{4}$     |
|           | $y = \frac{\pi}{4}$ |   |      |   | $I$              |   |      |   | $y = \frac{\pi}{4}$ |
| penne     | $0$                 | - | $-1$ | - | $-2$             | - | $-1$ | - | $0$                 |
| concavité | $0$                 | - | $-2$ | - | $0$              | + | $2$  | + | $0$                 |

### 8. Graphe de $f$

